Benha University **Faculty of Engineering-Shoubra** **Electrical Engineering Department** **3rd Year Computer Systems**		**Final Term Examination** **Automatic Control (Code: ECE312c)** **Date: 18 / 1 / 2016** **Duration: 3 hours**

- **Answer all the following questions** • **No. of questions : 5**
- **Illustrate your answers with sketches when necessary** • **Total Mark: 80 Marks**

***Question (1) (20 Marks) (Achieved ILOs;* a.1, a.2, b.2, b.3, b.4)**

a) A controlled system described by the differential equation: $y''(t) - 3y'(t) + 2y(t) = 8e^{2t}$

i. Find the solution of that system if *y(t)* and *x(t)* are the output and input of the system respectively and the initial condition are: $y(0) = y'(0) = 3$.

ii. Find the final value of *y(t).*

b) For the electrical circuit shown in Fig. (1):

i. Write the loop equations.

ii. Draw the block control diagram using these loop equations in (i).

iii. Find the transfer function $E_o(s)/E_i(s)$ as function of the electrical elements in the circuit.

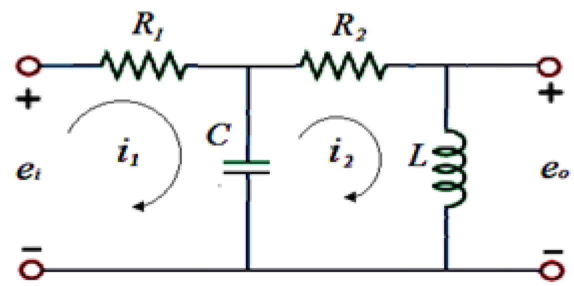


Fig. (1)

***Question (2) (20 Marks) (Achieved ILOs;* a.1, a.2, b.2, b.4)**

a) For the control system shown in Fig. (2); reduce that system to a single transfer function *C/R*.

b) For the signal flow graph shown in Fig. (3); find the transfer function *C(s)/R(s)* using Mason's rule.

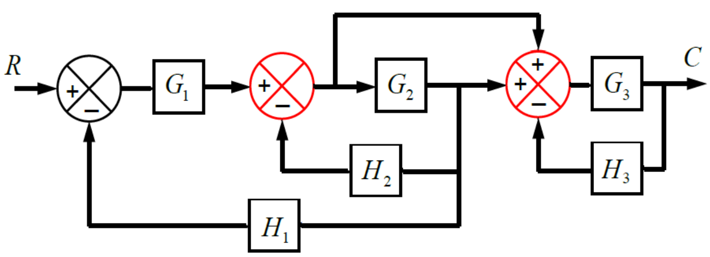


Fig. (2)

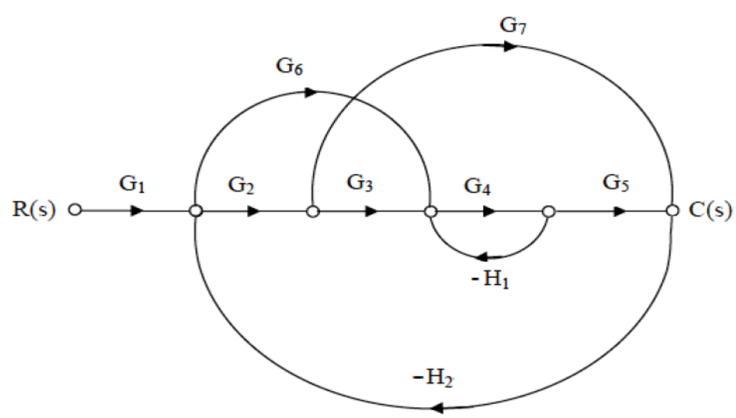


Fig. (3)

***Question (3) (15 Marks) (Achieved ILOs;* a.1, a.2, a4, b.5, b.16)**

a) For the position-control system shown in Fig. (3), determine the following.

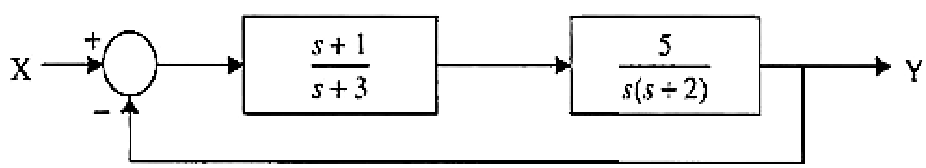


Fig. (3)

i. Find the position, velocity, and acceleration error constants.

ii. Find the steady-state error for a unit-step input, a unit-ramp input, and a unit-parabolic input.

b) Consider a second-order unity feedback system with $\zeta = 0.6$ and $\omega_n = 5$ rad/sec.

i. Calculate the rise time, peak time, maximum overshoot, and settling time when a unit-step input is applied to the system.

ii. Draw the response curve and show the damping type of the system

Question (4) (15 Marks) (Achieved ILOs;* a.1, a.2, a.4, b.3, b.11*)

a) Check the stability of the closed-loop system shown in Fig.(4).

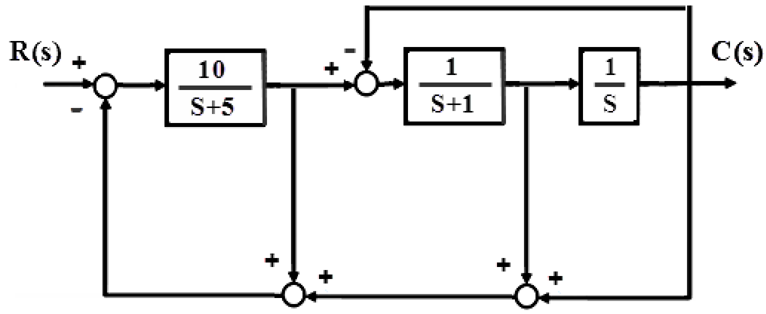


Fig.(4)

b) Consider the closed-loop system shown in Fig.(5). Determine the range of K for stability. Assume that K > 0.

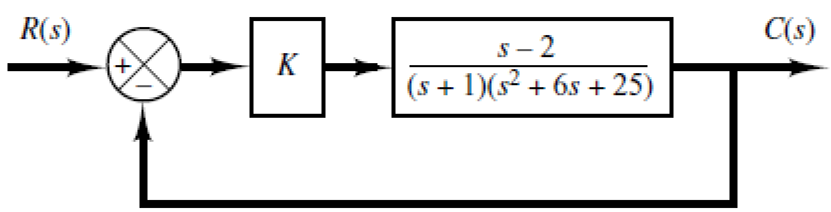


Fig.(5)

Question (5) (10 Marks) (Achieved ILOs;* a.1, a.2, b.2, b.16*)

For the digital control system shown in Fig.(6), **i)** Find the discrete-time output Y(z) of the closed-loop control system. **ii)** Obtain the continues-time output Y(s).

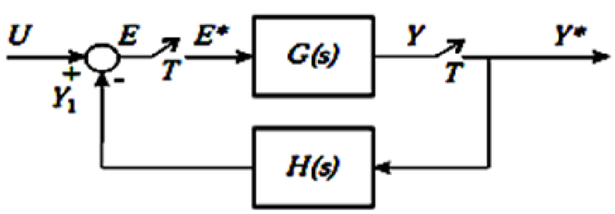


Fig.(6)

Model Answer

Question (1)

a) A controlled system described by the differential equation: $y''(t) - 3y'(t) + 2y(t) = 8e^{2t}$

i. Find the solution of that system if *y(t)* and *x(t)* are the output and input of the system respectively and the initial condition are: $y(0) = y'(0) = 3$.

ii. Find the final value of *y(t)*.

Answer

i. $y''(t) - 3y'(t) + 2y(t) = 8e^{2t}$

$$[S^2Y(s) - S\,y(0) - y'(0)] - 3[SY(s) - y(0)] + 2Y(s) = \frac{8}{S-2} \quad \text{............ if } y'(0) = y(0) = 3$$

$$[S^2Y(s) - 3S - 3] - 3[SY(s) - 3] + 2Y(s) = \frac{8}{S-2}$$

$$S^2Y(s) - 3S - 3 - 3SY(s) + 9 + 2Y(s) = \frac{8}{S-2}$$

$$S^2Y(s) - 3SY(s) + 2Y(s) - 3S + 6 = \frac{8}{S-2}$$

$$Y(s)(S^2 - 3S + 2) = \frac{8}{S-2} + 3S - 6$$

$$Y(s)[(S-1)(S-2)] = \frac{3(S-2)^2 + 8}{S-2}$$

$$Y(s) = \frac{3(S-2)^2 + 8}{(S-1)(S-2)^2} = \frac{3(S-2)^2}{(S-1)(S-2)^2} + \frac{8}{(S-1)(S-2)^2} = \frac{3}{(S-1)} + \frac{8}{(S-1)(S-2)^2}$$

$$\therefore \frac{3(S-2)^2 + 8}{(S-1)(S-2)^2} = \frac{3+A}{(S-1)} + \frac{B}{(S-2)^2} + \frac{C}{(S-2)}$$

First : *mutiply both sides by* $(S-1)$ *and lets* S *go to* $+1 \rightarrow \therefore A = 8$

Second : *mutiply both sides by* $(S-2)^2$ *and lets* S *go to* $+2 \rightarrow \therefore B = 8$

Third : *Put* $S = 0$ & $A = 8$ & $B = 8 \rightarrow \therefore C = -8$

$$\therefore \frac{3(S-2)^2 + 8}{(S-1)(S-2)^2} = \frac{11}{(S-1)} + \frac{8}{(S-2)^2} + \frac{-8}{(S-2)}$$

$$\therefore y(t) = 11e^{t} - 8e^{2t} + 8te^{2t}$$

ii.

$$\lim_{t\to\infty} y(t) = \lim_{S\to 0} SY(s) = \lim_{S\to 0} S\frac{3(S-2)^2 + 8}{(S-1)(S-2)^2} = 0$$

Question (1)

b) For the electrical circuit shown in Fig. (a):

i. Write the loop equations.

ii. Draw the block control diagram using these loop equations in (i).

iii. Find the transfer function $E_o(s)/E_i(s)$ as function of the electrical elements in the circuit.

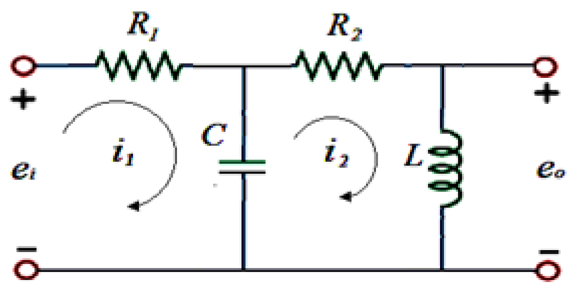


Fig. (a) t-domain circuit

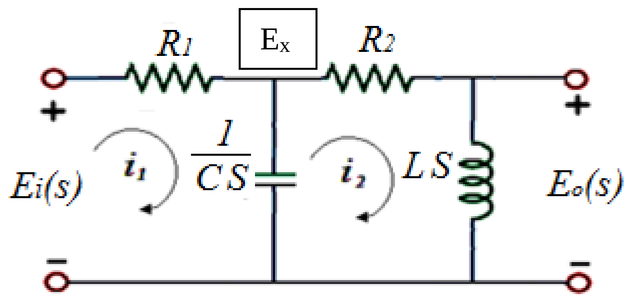


Fig. (b) S-domain circuit

Answer

i. From Fig. (b)

$$E_i(s) = I_1(s)[R_1 + \frac{1}{CS}] - I_2(s)\frac{1}{CS}$$

$$0 = I_2(s)[R_2 + \frac{1}{CS} + LS] - I_1(s)\frac{1}{CS}$$

$$E_o(s) = I_2(s)LS$$

ii. The control diagram is shown in Fig. (c)

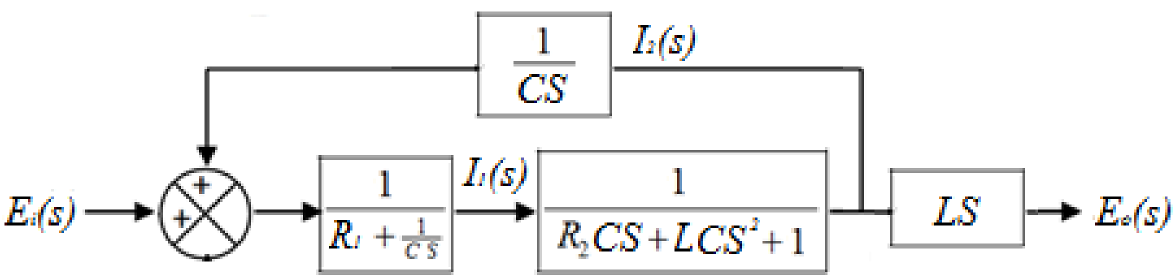


Fig. (c)

iii. From Fig. (b)

$$\frac{E_o(s)}{E_x(s)} = \frac{LS}{R_2 + LS} \text{ .... then ...... } E_o(s) = \frac{LS}{R_2 + LS}E_x(s) \text{ .... (1)}$$

$$Z = \frac{\left(\frac{1}{CS}\right)\left(R_2 + LS\right)}{\left(R_2 + LS\right)\left(\frac{1}{CS}\right)} = \frac{R_2 + LS}{R_2CS + LCS^2 + 1}$$

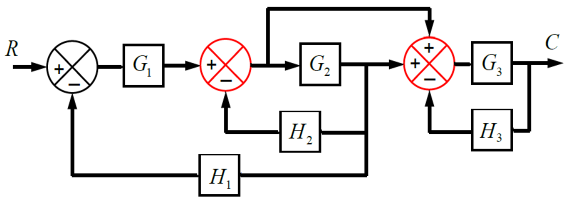
$$\frac{E_x(s)}{E_i(s)} = \frac{Z}{Z + R_1} \text{ .... then ...... } E_i(s) = E_x(s).\frac{R_1 + R_2 + LS + R_1LCS^2 + R_1R_2CS}{R_2 + LS} \text{ .... (2)}$$

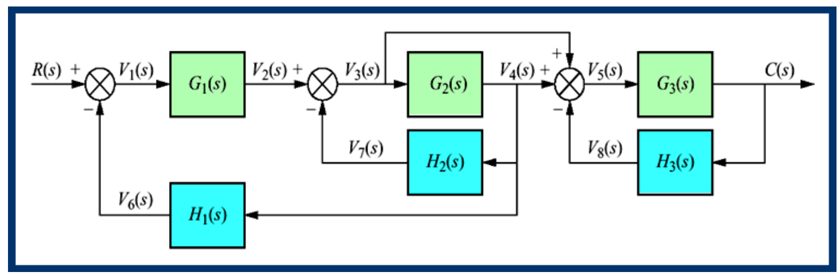
From Equations (1) & (2):

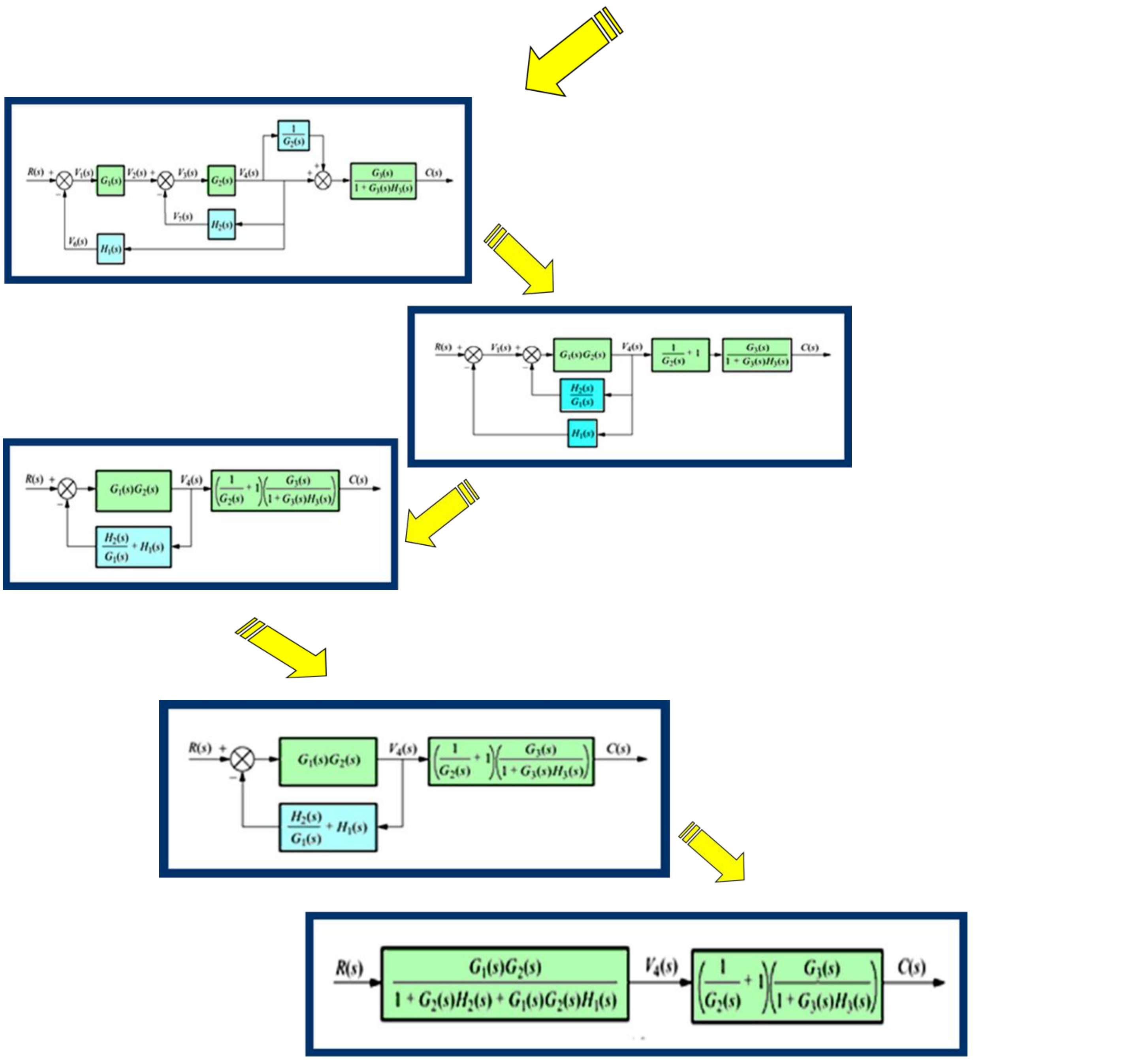
$$\frac{E_o(s)}{E_i(s)} = \frac{LS}{R_1 + R_2 + LS + R_1LCS^2 + R_1R_2CS}$$

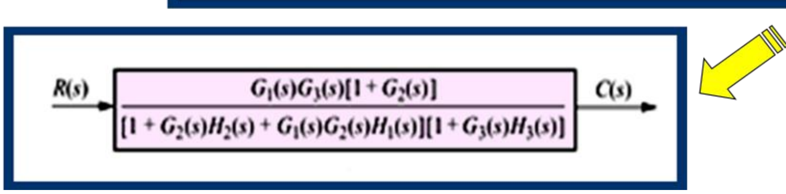
Question (2)

a) For the control system shown in Fig. (2); reduce that system to a single transfer function *C/R*.





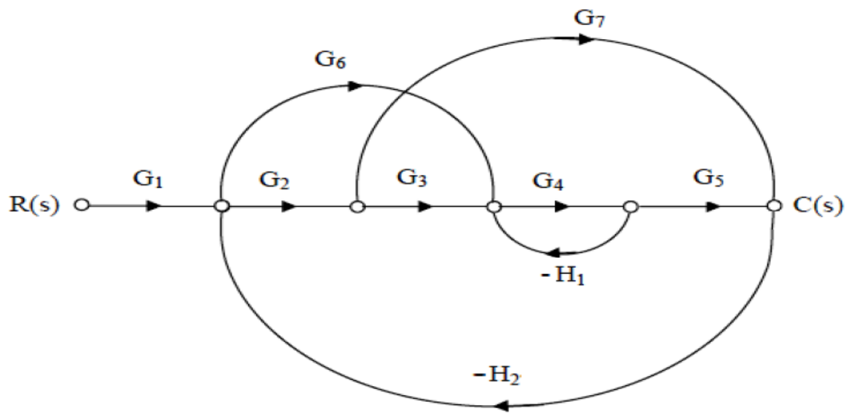




Ans. (3/9)

Question (2)

b) For the signal flow graph shown in Fig. (3); find the transfer function *C(s)/R(s)* using Mason's rule.



Solution

We have three forward path

$$P_1 = G_1G_2G_3G_4G_5$$
$$P_2 = G_1G_6G_4G_5$$
$$P_3 = G_1G_2G_7$$

We have three loops

$$L_1 = -G_4H_1$$
$$L_2 = -G_2G_7H_2$$
$$L_3 = -G_6G_4G_5H_2$$
$$L_4 = -G_2G_3G_4G_5H_2$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + L_1L_2$$

$$\Delta = 1 + G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_2G_3G_4G_5H_2 + G_2G_4G_7H_1H_2$$

All loops touching the forward paths P_1 ⇒ $\Delta_1 = 1$

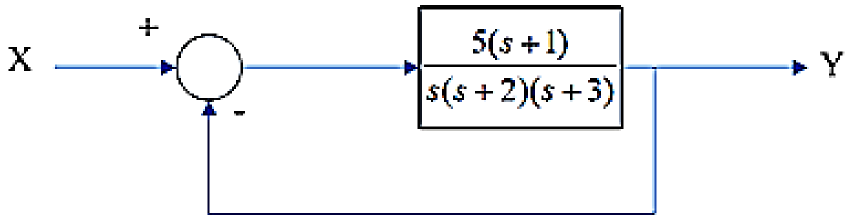
All loops touching the forward paths P_2 ⇒ $\Delta_2 = 1$

Only loop L_1 non touching the forward path P_3 ⇒ $\Delta_3 = 1 + G_4H_1$

$$G = \frac{1}{\Delta}\sum_{k=1}^{N} p_k\,\Delta_k \Rightarrow G = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2 + p_3\Delta_3}{\Delta}$$

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3G_4G_5 + G_1G_6G_4G_5 + G_1G_2G_7 + G_1G_2G_4G_7H_1}{1 + G_4H_1 + G_2G_7H_2 + G_6G_4G_5H_2 + G_2G_3G_4G_5H_2 + G_2G_4G_7H_1H_2}$$

3)- a)



$$G(s) = \frac{5(s+1)}{s(s+2)(s+3)}$$

Position error: $K_p = \lim_{s\to 0} G(s) = \lim_{s\to 0} \frac{5(s+1)}{s(s+2)(s+3)} = \infty$

Velocity error: $K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{5(s+1)}{(s+2)(s+3)} = \frac{5}{6}$

Acceleration error: $K_a = \lim_{s\to \infty} s^2G(s) = \lim_{s\to \infty} \frac{5s(s+1)}{(s+2)(s+3)} = 0$

Steady state error for unit step input:

$e_{ss} = \frac{1}{1+K_p}$

Referring to the result of problem 5-8, $K_p = \infty \Rightarrow e_{ss} = 0$

Steady state error for ramp input:

$e_{ss} = \frac{1}{K_v}$

Regarding the result of problem 5-8, $K_v = \frac{5}{6} \Rightarrow e(\infty) = \frac{6}{5}$

Steady state error for parabolic input:

$e_{ss} = \frac{1}{K_a}$

Regarding the result of problem 5-8, $K_a = 0 \Rightarrow e(\infty) = \infty$

3)- b)

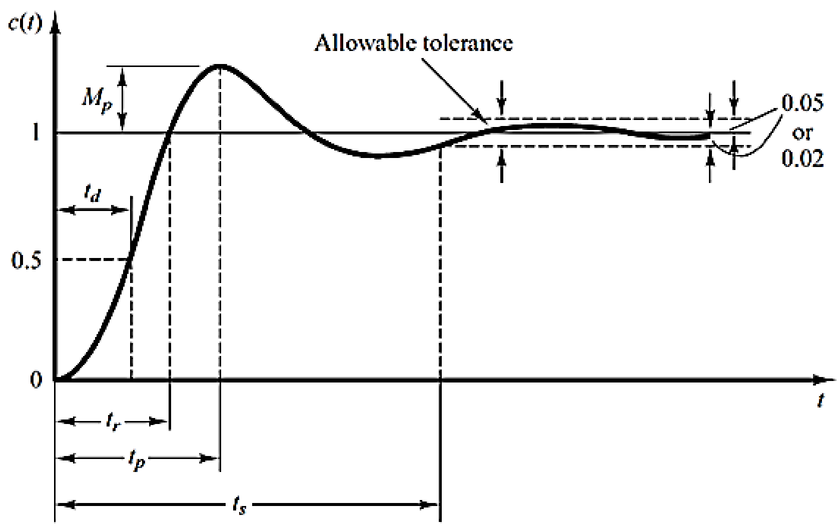
Rise time: $t_r \cong \frac{0.8+2.5\,\xi}{\omega_n} = \frac{0.8+2.5*0.6}{5} = 0.56\ sec$

Peak time: $t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{3.14}{5\sqrt{0.64}} = 0.785\ sec$

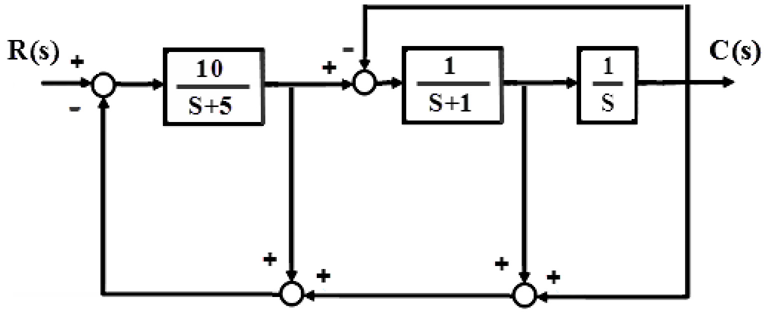
Maximum overshoot: $M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = e^{-\frac{0.6\pi}{0.8}} = 0.095$

Settling time: $t_s \cong \frac{3.2}{\xi\omega_n} \quad 0 < \xi < 0.69$

⇨ $t_s \cong \frac{3.2}{0.615} \cong 1.067\ sec$



4)- a)



The closed loop T.F. using Mason's rule is given by:

$$\frac{C}{R} = \frac{\frac{10}{s(s+1)(s+5)}}{1 - \left[-\frac{10}{s+5} - \frac{10}{(s+1)(s+5)} - \frac{10}{s(s+1)(s+5)} - \frac{1}{s(s+1)} \right]}$$

$$\frac{C}{R} = \frac{10}{s^3 + 16 s^2 + 26 s + 25}$$

The characteristic equation is:

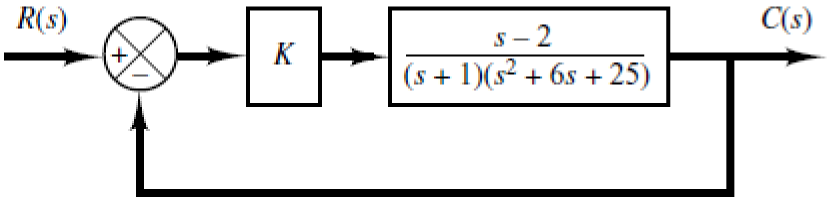
$S^3 + 16\ S^3 + 26\ S + 25 = 0$

Using Routh criterion method:

S^3	1	26
S^2	16	25
S^1	$\frac{16x26-1x25}{16} = 24.4$	0
S^0	$\frac{24.4x25-16x0}{24.4} = 26$	0

The sign in the first column is not changed. Then, the system is stable

4)- b)



The closed-loop transfer function C(s)/R(s) is

$$\frac{C(s)}{R(s)} = \frac{K(s-2)}{(s+1)(s^2+6s+25)+K(s-2)}$$

$$= \frac{K(s-2)}{s^3+7s^2+(31+K)s+25-2K}$$

For stability, the denominator of this last equation must be a stable polynomial. For the characteristic equation

$$s^3+7s^2+(31+K)s+25-2K=0$$

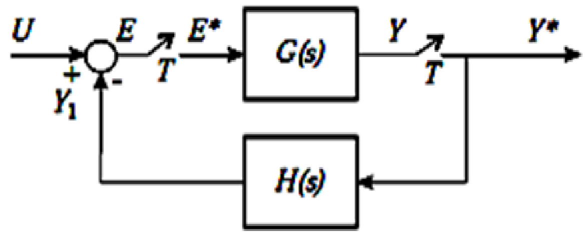
the Routh array becomes as follows:

$$\begin{array}{lll} s^3 & 1 & 31+K \\ s^2 & 7 & 25-2K \\ s^1 & \frac{192+9K}{7} & 0 \\ s^0 & 25-2K & \end{array}$$

Since K is assumed to be positive, for stability, we require

$$12.5 > K > 0$$

5)



$$E(s) = U(s) - Y_1 = U(s) - H(s)Y^*(s)$$

$$Y(s) = G(s)E^*(s)$$

taking the Pulse transfer for the above equ

$$E^*(s) = U^*(s) - [H(s)Y^*(s)]^*$$

$$= U^*(s) - H^*(s)Y^*(s)$$

$$Y^*(s) = [G(s)E^*(s)]^* = G^*(s)E^*(s)$$

$$\therefore Y^*(s) = G^*(s)[U^*(s) - H^*(s)Y^*(s)]$$
$$= G^*(s) \cdot U^*(s) - G^*(s)H^*(s)Y^*(s)$$

$$Y^*(s) = \frac{G^*(s)U^*(s)}{1 + G^*(s)H^*(s)} \quad \#$$

now we can deduce Z-transform notat the discrete-time o/P :-

$$Y(z) = \frac{G(z)U(z)}{1+G(z)H(z)} \quad \# \ (i)$$

$$Y(z) = \frac{G(z)U(z)}{1+G(z)H(z)}$$

ii) Continous-Time output :-

$$Y(s) = G(s)E^*(s)$$

$$= G(s)[U^*(s) - H^*(s)Y^*(s)]$$

$$= G(s)U^*(s) - G(s)H^*(s)Y^*(s)$$

$$= G(s)U^*(s) - G(s)H^*(s) \cdot \frac{G^*(s)U^*(s)}{1+G^*(s)H^*(s)}$$

$$Y(s) = G(s)U^*(s)\left[1 - \frac{G^*(s)H^*(s)}{1+G^*(s)H^*(s)}\right]$$

$\#$ (ii)